Predicate Viability and Operator Collapse in the UNNS Framework

1 Introduction

Mathematical practice routinely distinguishes between questions that are *answered*, questions that are *open*, and questions that are *undecidable* within a given axiomatic system. However, a fourth category is often left implicit: questions whose predicates are applied to structures for which the predicates themselves are not structurally admissible.

Classical number theory, for example, treats all natural numbers as equally eligible for arithmetical predicates such as divisibility or primality. When confronted with extreme but finitely defined quantities—such as TREE(3) or Graham's number—this treatment leads to statements of the form "the answer exists but is unknowable in principle". Such statements conflate epistemic limitation with structural applicability.

The UNNS framework approaches this issue from a different direction. Rather than asking whether a predicate can be computed or proven, it asks whether the predicate is *meaningfully applicable* to the structure in question. This determination is made structurally, by analyzing how recursive objects behave under successive regimes of generability, internal consistency, and closure.

The central claim of this paper is that predicate applicability is not universal. Certain predicates presuppose structural invariants that do not survive projection and collapse. When these invariants fail to persist, the predicates are not merely undecidable or unknown, but structurally non-applicable.

To formalize this distinction, we introduce the notion of *predicate viability*. A predicate is viable for a given structure if and only if the structural information required for its evaluation survives the progression through the $\Phi \to \Psi \to \tau$ regimes and remains invariant under collapse. Failure of viability is not a limitation of computation, proof, or observation; it is a property of the structure itself under admissible projection.

This paper develops a minimal axiomatic framework for predicate viability and applies it to canonical examples. In particular, we show that while TREE(3) is finitely defined and mathematically valid, it fails to admit the structural invariants required for primality to be applicable. Accordingly, the primality predicate is non-viable for TREE(3) within the UNNS framework.

The analysis deliberately avoids claims about truth, existence, or empirical observability. As in prior UNNS work, the distinction between intrinsic structure and admissible description is maintained throughout. The results presented here concern the conditions under which statements about structures are meaningful, not the ontological status of the structures themselves.

The remainder of the paper is organized as follows. Section 2 introduces the UNNS structural regimes and the notions of Ψ-stable reduction, admissible projection, and τ-level invariance. Section 3 states the predicate viability axiom and derives its immediate consequences. Section 4 applies the framework to representative examples, including TREE(3). Section 5 discusses the relationship between non-viability, undecidability, and classical notions of mathematical ignorance.

2 Operator Regimes and Structural Admissibility

This section introduces the minimal formal apparatus required to state predicate viability within the UNNS framework. The goal is not to exhaustively characterize recursive structure, but to specify the conditions under which predicates may be meaningfully applied to mathematical objects under projection and collapse.

Throughout, we distinguish *structural regimes* from *operators*. The symbols Φ, Ψ, and τ label successive regimes of organization of recursive structure. They do not denote actions performed on objects, but rather stages at which different forms of structural coherence may or may not persist. Operator XII (Collapse) acts subsequently as a destructive filter and is not identified with any of these regimes.

2.1 Structural Regimes

Definition 1 (Structural Regimes)**.** Let X be a recursive structure. The UNNS framework distinguishes three structural regimes:

- Φ (*Generative Regime*): the regime of finite symbolic or combinatorial generability. A structure is Φ-admissible if it is specified by a finite rule or description.

- Ψ (*Structural Regime*): the regime of relational consistency. A structure is Ψ-admissible if it admits stable reductions, symmetries, or relational descriptions that preserve its identity under refinement.
- τ (*Closure Regime*): the regime of irreducible closure. A structure is τ-admissible if it exhibits invariant structure that persists under admissible projection and collapse.

Regime admissibility is monotone: failure at any regime prevents admissibility at subsequent regimes.

The collapse operator (Operator XII), introduced later, acts only after regime progression and does not constitute an additional regime.

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2.2 Ψ-Stable Reductions

Definition 2 (Ψ-Stable Reduction)**.** Let X be a Φ-admissible structure. A reduction

$$\rho : X \to X'$$

is called Ψ*-stable* if it satisfies the following conditions:

1. *Termination*: the reduction completes in finitely many steps.
2. *Invariant Preservation*: there exists a structural invariant I such that
$$I(X) = I(\rho(X)).$$
3. *Stabilization*: repeated application converges, i.e. there exists n such that
$$\rho^n(X) = \rho^{n+1}(X).$$

A structure is Ψ-admissible if it admits at least one Ψ-stable reduction.

Examples. Typical examples of Ψ-stable reductions include:

- prime factorization of finite integers, preserving multiplicative structure;
- continued fraction expansion of quadratic irrationals, preserving approximation class;
- symmetry quotients in geometry, preserving relational identity;

- modular reduction for objects with intrinsic periodic structure.

Structures that admit no such reduction lack analyzable internal organization at the Ψ-regime.

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2.3 Admissible Projections

Definition 3 (Admissible Projection)**.** Let X be a Ψ-admissible structure. A projection

$$\pi : X \to \widehat{X}$$

is called *admissible* if it satisfies:

1. *Coarse-Graining*: π reduces resolution while preserving structural identity.
2. *Interaction Preservation*: for any admissible binary operation $\oplus$,

$$\pi(x \oplus y) \sim \pi(x) \mathbin{\widehat{\oplus}} \pi(y),$$

   where $\sim$ denotes structural equivalence.
3. *Stability*: small perturbations of X do not destroy the projected structure relevant to predicate evaluation.

Examples. Admissible projections include:

- digit truncation preserving order of magnitude;
- modular projection preserving residue classes;
- sampling or discretization preserving frequency or relational content.

Projections that destroy the structural features required for predicate evaluation are not admissible.

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2.4 τ-Level Invariants

Definition 4 (τ-Invariant)**.** Let X be a Ψ-admissible structure. A structural invariant $I(X)$ is called *τ-level* if:

1. *Projection Stability*: $I(X)$ is preserved under all admissible projections π.

2. *Interaction Meaningfulness*: I participates nontrivially in the evaluation of one or more predicates.

3. *Reduced Determinability*: $I(X)$ can be determined from $\pi(X)$ for admissible projections.

A structure admits τ-closure if it possesses at least one τ-level invariant.

Examples. Typical τ-level invariants include:

- parity of integers (stable under modulo-2 projection);
- sign and order class (stable under coarse magnitude projection);
- closure constraints such as $\sqrt{2}$, e, or π in their respective regimes.

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2.5 Collapse and Predicate Admissibility

Operator XII (Collapse) acts as a destructive filter eliminating structures that fail to maintain τ-closure. It does not generate invariants, but reveals those that persist by eliminating all others.

Predicates whose evaluation presupposes τ-level invariants are admissible only for structures that survive collapse. This principle underlies the predicate viability axiom introduced in the next section.

This separation between intrinsic structure and admissible description parallels the distinction between τ-closure and empirical observability established in the τ-closure observability framework, where failure of detectability constrains permissible statements without bearing on substrate validity.

3 Predicate Viability Axiomatics

We now formalize the criterion governing when a predicate may be meaningfully applied to a recursive structure within the UNNS framework. The guiding principle is that predicates are not universally applicable; their admissibility depends on the survival of specific structural invariants through the UNNS regime progression and collapse.

Throughout this section, predicates are treated as abstract evaluative mappings rather than as computational procedures. No assumption is made regarding decidability, efficiency, or provability.

3.1 Predicate Viability

Axiom 1 (Predicate Viability under τ-Closure)**.** *Let X be a recursive structure admissible at the Φ-regime. A predicate P is said to be* viable *for X if and only if the evaluation of P depends exclusively on structural invariants that persist through the $\Phi \to \Psi \to \tau$ regime progression and remain invariant under the action of Operator XII (Collapse).*

If X fails to admit τ-closure, then any predicate whose evaluation presupposes τ-level invariance is not admissible for X.

This axiom does not assert the truth or falsity of predicates. It specifies the conditions under which predicates are structurally admissible.

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3.2 Monotonicity of Regime Failure

Lemma 1 (Operator Monotonicity)**.** *Let X be a recursive structure. If X fails to be admissible at some regime $R \in \{\Phi, \Psi, \tau\}$, then X is inadmissible at all subsequent regimes.*

Proof. By definition, Ψ-admissibility presupposes Φ-admissibility, and τ-admissibility presupposes Ψ-admissibility. Failure at any regime eliminates the structural prerequisites required for admissibility at later regimes. □

This monotonicity ensures that predicate failure may be localized to the earliest regime at which the necessary structure collapses.

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3.3 Non-Viability versus Undecidability

Predicate non-viability must be distinguished from classical notions of undecidability.

A predicate may be:

- *Viable but undecided*, when the predicate is structurally admissible but its truth value is not known;
- *Undecidable*, when the predicate is admissible but unresolvable within a given formal system;
- *Non-viable*, when the predicate fails to be structurally admissible due to regime collapse.

Undecidability reflects limitations of formal systems. Non-viability reflects structural inapplicability under projection and collapse. These notions are logically independent.

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3.4 Scope of the Axiom

The predicate viability axiom does not eliminate mathematical uncertainty. It excludes only predicates whose applicability presupposes invariants that do not survive the UNNS regime progression.

In particular, the framework permits:

- open problems with well-defined predicates;
- undecidable but meaningful questions;
- context-dependent predicates whose viability depends on projection.

The axiom therefore refines, rather than replaces, classical classifications of mathematical questions.

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3.5 Relation to Observability

The distinction between predicate viability and predicate evaluation parallels the distinction between intrinsic τ-closure and empirical observability established in the τ-closure observability framework. In both cases, failure constrains admissible statements without bearing on the existence or internal consistency of the underlying structure.

4 Worked Examples

We now apply the predicate viability framework to representative mathematical objects. The purpose of these examples is not to exhaustively classify mathematical structures, but to illustrate the distinct modes of predicate success and failure under the $\Phi \to \Psi \to \tau$ regime progression.

In particular, we contrast:

- finite but extreme combinatorial quantities,
- finitely defined but structurally opaque numbers,
- algebraic quantities admitting stable closure.

4.1 Summary Table

Table 1 summarizes the regime admissibility and predicate viability outcomes for the examples considered in this section.

Object	Φ	Ψ	τ	**Predicate Outcome**
TREE(3)	✓	×	×	Primality non-viable
Graham's number G	✓	×	×	Primality non-viable
$\sqrt{2}$	✓	✓	✓	Rationality viable

Failure at a regime indicates structural inadmissibility, not epistemic limitation.

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4.2 Example 1: TREE(3)

The quantity TREE(3) arises from a finite combinatorial game definition and is a well-defined natural number. Its magnitude exceeds that of any quantity encountered in conventional mathematics, but its definition is finite and unambiguous.

Regime Analysis.

- **Φ-admissibility.** TREE(3) is generated by a finite symbolic rule and is therefore admissible at the Φ-regime.
- **Ψ-failure.** No Ψ-stable reduction preserving divisor interaction is available. In particular, no terminating reduction exists that preserves multiplicative structure or yields a stable relational decomposition.
- **τ-failure.** Since Ψ-admissibility fails, no invariant structure persists under admissible projection and collapse. Consequently, TREE(3) does not admit τ-closure.

Predicate Consequence. The predicate of primality presupposes τ-level invariance of divisor structure. Since such invariants do not survive collapse for TREE(3), the primality predicate is not admissible. This failure reflects structural non-viability rather than ignorance or infeasibility.

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4.3 Example 2: Graham's Number

Graham's number G is defined by a finite expression using Knuth's up-arrow notation. Like TREE(3), it is a legitimate natural number whose magnitude vastly exceeds practical representation.

Regime Analysis.

- **Φ-admissibility.** Graham's number is specified by a finite recursive description and is therefore admissible at the Φ-regime.
- **Ψ-failure.** Despite its finite definition, G admits no Ψ-stable reduction preserving arithmetic interaction. There exists no relational decomposition, symmetry quotient, or terminating reduction stabilizing under iteration.
- **τ-failure.** In the absence of Ψ-stability, admissible projections eliminate all structure required for arithmetic predicates. No τ-level invariant persists.

Predicate Consequence. As with TREE(3), the primality predicate is not viable for Graham's number. The framework therefore classifies primality questions about G as structurally inadmissible rather than undecidable.

This demonstrates that extreme magnitude alone is not the determining factor; the failure arises from the absence of analyzable structure.

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4.4 Example 3: $\sqrt{2}$

The quantity $\sqrt{2}$ is defined as the positive solution to $x^2 = 2$ and represents a paradigmatic algebraic irrational.

Regime Analysis.

- **Φ-admissibility.** $\sqrt{2}$ is finitely specified via a polynomial equation and is admissible at the Φ-regime.
- **Ψ-admissibility.** $\sqrt{2}$ admits stable relational structure, including its periodic continued fraction expansion. These reductions terminate in a stable equivalence class.
- **τ-admissibility.** The invariant distinguishing rational from irrational quantities persists under admissible projections. The irrationality of $\sqrt{2}$ survives coarse-graining and collapse.

Predicate Consequence. The predicate "is rational" is viable for $\sqrt{2}$. Accordingly, the statement "$\sqrt{2}$ is irrational" is meaningful and evaluable within the UNNS framework.

This example demonstrates that the framework preserves classical mathematical judgments when the required structural invariants survive.

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4.5 Interpretive Summary

These examples illustrate three distinct outcomes:

- Φ-admissible but Ψ- and τ-inaccessible structures, for which arithmetic predicates are non-viable;
- finite definability does not guarantee predicate applicability;
- classical results are recovered when structural invariants persist.

Predicate non-viability thus identifies a structural boundary on meaningful mathematical statements, distinct from undecidability or lack of proof.

5 Classical and UNNS Classifications of Mathematical Questions

Classical mathematics distinguishes primarily between questions that are resolved, open, or undecidable within a given formal system. While effective for many purposes, this classification conflates distinct structural failure modes. In particular, it does not distinguish between predicates that are meaningful but unresolved and predicates that are structurally inapplicable to the objects under consideration.

The UNNS framework introduces an additional layer of classification by separating predicate *viability* from predicate *evaluation*. This section contrasts the classical and UNNS perspectives and clarifies the distinction between non-viability and undecidability.

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5.1 Classical versus UNNS Perspective

Table 2 contrasts how representative objects and questions are treated in classical mathematics and under the UNNS predicate viability framework.

Object / Question	Classical Status	UNNS Classification
17 prime?	Resolved	Predicate viable, evaluated
TREE(3) prime?	Unknown in principle	Predicate non-viable
Graham's number prime?	Unknown in principle	Predicate non-viable
$\sqrt{2}$ rational?	Resolved	Predicate viable, evaluated
Goldbach conjecture	Open	Predicate viable, unevaluated
Continuum hypothesis	Undecidable (ZFC)	Predicate viable, framework-relative

In the UNNS framework, "unknown in principle" is not a primitive category. Questions previously grouped under this label are separated according to whether their predicates are structurally admissible.

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5.2 Non-Viability versus Undecidability

Predicate non-viability must be sharply distinguished from undecidability.

Undecidability. A predicate is undecidable when it is structurally admissible but cannot be resolved within a given formal system. Undecidability reflects limitations of axiomatic frameworks and proof systems.

Non-Viability. A predicate is non-viable when the structural invariants required for its evaluation do not survive the $\Phi \to \Psi \to \tau$ regime progression. In this case, the predicate is not meaningfully applicable, independent of any formal system.

These notions are logically independent. A predicate may be viable yet undecidable, or non-viable without invoking any formal incompleteness.

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5.3 Classification of Question Types

The distinction may be summarized as follows:

Question Type	Classical View	UNNS View
Resolved	Answer known	Viable, evaluated
Open	Answer unknown	Viable, unevaluated
Undecidable	No proof in system	Viable, unresolvable in framework
Non-viable	Not distinguished	Predicate structurally inadmissible
Category error	Informal notion	Predicate inapplicable by type

Classical mathematics typically treats non-viable predicates as extreme instances of open or undecidable questions. UNNS instead identifies them as failures of structural applicability.

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5.4 Implications for Mathematical Practice

The introduction of predicate viability refines, rather than replaces, classical mathematical reasoning. It preserves existing results where structural invariants persist, while preventing misclassification of questions whose predicates do not survive collapse.

In particular, the framework:

- preserves open problems with meaningful predicates;
- respects undecidability results without reinterpretation;
- excludes structurally inapplicable predicates without appeal to epistemic limitation.

The distinction between non-viability and undecidability therefore clarifies the logical status of questions involving extreme or opaque structures, without altering the foundations of classical mathematics.

6 Physical Predicate Viability

The predicate viability framework developed above applies not only to mathematical structures, but also to physical quantities insofar as they are represented by mathematical structures subject to projection, interaction, and collapse.

In physical theories, predicates typically correspond to statements of observability, measurability, or dynamical relevance. As in the mathematical case, such predicates presuppose the persistence of specific structural invariants. The UNNS framework makes this dependence explicit.

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6.1 Observability as Predicate Viability

In physical contexts, a predicate is viable if and only if the quantity it refers to admits τ-level invariants under admissible physical projection. Here, admissible projections correspond to experimentally realizable coarse-grainings, such as finite resolution, noise, or indirect measurement.

This aligns with the τ-closure observability framework, where observability is not identified with existence, but with the persistence of invariant structure under collapse.

[Physical Predicate Viability] A physical predicate referring to a quantity Q is viable if and only if the structural representation of Q admits τ-closure under admissible physical projections.

Failure of viability indicates that no meaningful physical statement can be formed, independent of experimental limitation.

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6.2 Representative Physical Examples

We illustrate the correspondence using representative physical quantities.

Position. The position of a particle admits stable projection under finite spatial resolution. Relational structure is preserved under coarse-graining, and positional predicates remain viable. Accordingly, position is a τ-viable observable.

Momentum. Momentum admits dual representation via Fourier structure. Despite projection into finite bandwidth or resolution, invariant relational structure persists. Momentum predicates are therefore τ-viable.

Weinberg Angle. The Weinberg angle emerges as a stable relational invariant across renormalization scales. Its persistence under projection and effective field descriptions indicates τ-closure. Predicates referring to its value are viable and meaningful. This behavior has been explicitly tested within the UNNS Chamber system.

Planck-Scale Couplings. Quantities defined only at trans-Planckian scales do not admit admissible projection into experimentally accessible regimes. No τ-level invariant structure persists. Predicates referring to precise values of such quantities are therefore non-viable within the framework.

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6.3 Connection to the UNNS Chamber System

The UNNS Chamber system provides an experimental and computational environment for testing predicate viability across regimes.

In particular:

- Chamber XIII examines the τ-viability of electroweak parameters, including the Weinberg angle.
- Chamber XXXII investigates τ-closure through spectral observability gates.
- Chambers XIX and XX explore the emergence of Maxwell-type structure under recursive projection.

These chambers do not establish physical truth. They test whether the structural invariants required for predicate viability persist under admissible collapse.

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6.4 Interpretive Boundary

The extension of predicate viability to physical quantities does not assert that non-viable predicates correspond to non-existent entities. As in the mathematical case, non-viability reflects structural inadmissibility, not ontological absence.

This preserves the separation between the UNNS Substrate, in which physical structure may exist, and the UNNS Framework, which governs which statements about that structure are meaningful.

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6.5 Summary

Physical predicate viability parallels mathematical predicate viability. In both domains, meaningful statements require the persistence of invariant structure under projection and collapse. UNNS therefore provides a unified criterion for distinguishing meaningful physical observables from structurally inadmissible predicates, without appeal to epistemic or experimental limitation.

Remark 1 (Framework–Substrate Separation). The UNNS Substrate denotes the underlying recursive structural domain in which generability, consistency, and closure properties are defined intrinsically. The UNNS Framework is the formal apparatus—operators, regimes, axioms, and admissibility criteria—used to articulate which statements about substrate-level structures are meaningful under projection and collapse.

Results established within the UNNS Framework do not assert the presence or absence of properties within the Substrate itself. Rather, they determine the viability of predicates under the action of the $\Phi \to \Psi \to \tau$ regime progression and the destructive filtering of Operator XII.

Accordingly, the conclusion that primality is non-viable for TREE(3) is a framework-level statement concerning predicate admissibility after collapse. It does not constitute an ontological claim about the internal structure of the Substrate, nor does it appeal to computational, epistemic, or observational limitation. This distinction parallels the separation between intrinsic τ-closure and empirical observability established in the τ-closure observability framework, where the failure of detectability constrains admissible statements without bearing on substrate validity.

```
      X
      |
     []  -- fail --> non-viable
      |
     []  -- fail --> non-viable
      |
     [τ]  -- fail --> non-viable
      |
 predicate viable
```

Figure 1: Operator cascade determining predicate viability. Failure at any regime renders predicates requiring subsequent structure inadmissible.

```
TREE(3)
   |

   |

   |
  τ
   |
primality: non-viable
```

Figure 2: Regime trajectory for TREE(3). Structural collapse at the Ψ regime prevents τ-closure and renders primality inadmissible.

```
predicate complexity
^
|   viable
|
|
|           non-viable
|
+------------------------> object complexity
```

Figure 3: Schematic viability landscape. Predicate applicability depends jointly on object structure and predicate requirements.

7 Conclusion and Discussion

This paper introduced a structural criterion for determining when predicates are meaningfully applicable to mathematical and physical structures. By separating predicate viability from predicate evaluation, the UNNS framework distinguishes between questions that are unresolved, undecidable, and structurally inadmissible.

The central contribution is the identification of *non-viability* as a distinct failure mode. Unlike undecidability, which reflects limitations of formal systems, non-viability arises from the absence of invariant structure required for predicate application. This distinction resolves long-standing ambiguities surrounding extreme but finitely defined objects, such as TREE(3) and Graham's number, without appealing to epistemic or computational limits.

The worked examples demonstrate that UNNS preserves classical mathematical judgments whenever structural invariants persist. Well-established results, such as the irrationality of $\sqrt{2}$, remain intact. Open problems with meaningful predicates, such as Goldbach's conjecture, are unaffected. Only predicates whose structural prerequisites fail under the $\Phi \to \Psi \to \tau$ regime progression are excluded.

The extension to physical quantities shows that the same criterion governs observability. Physical predicates are viable precisely when invariant structure survives admissible projection and collapse. This provides a unified account of mathematical and physical meaningfulness while maintaining a strict separation between the UNNS Substrate and the UNNS Framework.

Several directions remain open. A systematic classification of predicates by their regime requirements would further clarify the boundary between viable and non-viable questions. Connections to complexity theory, while suggestive, require careful formulation to avoid conflation of structural and computational notions. Finally, the Chamber system offers a practical environment for testing predicate viability across domains.

In summary, UNNS reframes foundational questions by replacing appeals to unknowability with structural analysis. Predicate applicability becomes a property of recursive structure under collapse, not a matter of epistemic reach.

A Formal Proofs

This appendix provides formal justification for the structural claims used in the main text. The proofs are not intended to establish numerical bounds, but to demonstrate failure of predicate admissibility under the UNNS regime criteria.

A.1 Failure of Primality Admissibility for TREE(3)

[Primality Non-Viability for TREE(3)] The predicate of primality is not admissible for TREE(3) within the UNNS framework.

Proof. By construction, TREE(3) is generated by a finite combinatorial rule and is therefore admissible at the Φ-regime.

Assume, for contradiction, that the primality predicate is admissible for TREE(3). Then there must exist a τ-level invariant encoding divisor interaction that persists under admissible projection.

Such an invariant would presuppose the existence of a Ψ-stable reduction preserving multiplicative structure, i.e. a terminating relational decomposition that stabilizes under iteration and is preserved under projection.

However, any divisor-based reduction for TREE(3) requires interaction with structure at a scale proportional to TREE(3) itself. No terminating reduction preserving divisor structure stabilizes under iteration, nor does any admissible projection retain information sufficient to reconstruct divisor interaction.

Thus, TREE(3) fails to admit Ψ-stable reduction and consequently fails to admit τ-closure. This contradicts the assumed admissibility of the primality predicate.

Therefore, primality is not admissible for TREE(3). □

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A.2 Monotonicity of Regime Failure

[Operator Monotonicity] Let X be a recursive structure. If X fails admissibility at some regime $R \in \{\Phi, \Psi, \tau\}$, then X fails admissibility at all subsequent regimes.

Proof. By definition, Ψ-admissibility presupposes Φ-admissibility, and τ-admissibility presupposes Ψ-admissibility. Failure at regime R eliminates the structural prerequisites required for admissibility at any subsequent regime. □

These results justify the localization of predicate failure to the earliest regime at which structural admissibility collapses.